Test of the QCD vacuum with the sources in higher representations

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Abstract

Recent accurate measurement [1] [2] of static potentials between sources in various $SU(3)$ representations provides a crucial test of the QCD vacuum and of different theoretical approaches to the confinement. In particular, the Casimir scaling of static potentials found for all measured distances implies a strong suppression of higher cumulants and a high accuracy of the Gaussian stochastic vacuum. Most popular models are in conflict with these measurements.

1. An accurate measurement of static potentials between sources in the eight different representations of $SU(3)$ group made recently in [1] reveals a new quantitative picture of the QCD vacuum and provides a crucial test of existing theoretical models. Other measurements of static interaction [2] are in general agreement with [1].

The most useful way to represent static potentials $V_D(r)$ in representations $D = 3, 8, 6, 15a, 10, 27, 24, 15s$ is through the complete set of field correlators in the framework of the Field Correlator Method (FCM) [3]:

$$V_D(r) = -\lim_{T\to\infty} \frac{1}{T} \ln\langle W(C)\rangle, \qquad (1)$$

where Wilson loop $W(C)$ for the rectangular contour $C = r \times T$ in the (34) plane has the cumulant expansion,

$$\langle W(C)\rangle = Tr_D exp \int_S \sum_{n=2,4...} (ig)^n \langle\langle F(1)F(2)..F(n)\rangle\rangle d\sigma(1)...d\sigma(n) \qquad (2)$$

Here $F(k)d\sigma(k) = F_{34}(u^{(k)}, x_0)d\sigma_{34}(^{(k)})$ and the component $F_{34}(u, x_0) \equiv E_3(u, x_0) = \phi(x_0, u)E_3(u)\phi(u, x_0)$, where ϕ is a parallel transporter and x_0 is an arbitrary point on the surface S inside contour C; $Tr_D \hat{1} = 1$.

Dependence on D enters in (2) through the generators T^a, since $F(k) = F^a(k)T^a(a = 1, ...N_c^2 - 1)$, and the main characteristics of D is the quadratic Casimir operator C_D: $T^aT^a = \hat{1}C_D$, so that the invariant square of the color charge in the representation D is g^2C_D.

One can now express the connected correlators (cumulants) in (2) via C_D and D–independent averages as follows (for more details see last reference in [3] and[4]),

$$Tr_D\langle F(1)F(2)\rangle = C_D\frac{\langle F^a(1)F^a(2)\rangle}{N_c^2 - 1}, \tag{3}$$

$$\begin{aligned} Tr_D\langle\langle F(1)F(2)F(3)F(4)\rangle\rangle = \frac{C_D^2}{(N_c^2 - 1)^2}\{\langle F^a(1)F^a(2)F^b(3)F^b(4)\rangle\rangle \\ +\langle F^a(1)F^b(2)F^b(3)F^a(4)\rangle - \langle F^a(1)F^a(2)\rangle\langle F^b(3)F^a(4)\rangle \\ + (1 - \frac{N_c}{2C_D})\langle F^a(1)F^b(2)F^a(3)F^b(4)\rangle\} + O(\frac{1}{N_c^2}). \end{aligned} \tag{4}$$

Note that the arguments of $F(k)$ in (4) and in (2) are ordered (e.g. clockwise, $u^{(1)} < u^{(2)} < u^{(3)} < u^{(4)}$) and therefore the only vacuum insertion is possible in the first term on the r.h.s. of (4) leading to the cancellation with the third term: hence the correlator (4) is a connected one vanishing at large distances, $|u^{(1)} + u^{(2)} - u^{(3)} - u^{(4)}| \to \infty$.

One can show in a similar way that the n-th cumulant in (2) contributes proportionally to C_D^n. As a result the static potential $V_D(r)$ has the expansion

$$V_D(r) = d_DV^{(2)}(r) + d_D^2V^{(4)}(r) + ..., \tag{5}$$

where in notations of ref. [1] $d_D = C_D/C_F$ and C_F is the fundamental Casimir operator, $C_F = \frac{N_c^2-1}{2N_c}$. The fundamental static potential contains perturbative Coulomb part V_{Coul}, confining linear and constant terms.

The Coulomb part ,which is also obtainable from the perturbative component of the FC in (3) is now known up to two loops [5] and is proportional to C_D. Therefore one may expect quartic contributions proportional to $C_D^2 \sim d_D^2$, to the constant and linear terms, writing (5) as

$$V_D(r) = d_DV^{(2)}(r) + d_D^2(\bar{v}_0^{(4)} + \bar{\sigma}_4 r). \tag{6}$$

Here $\bar{v}_0, \bar{\sigma}_4$ measure the contribution of the quartic cumulants to the constant term and string tension respectively.

Now the measurements of ref. [1] allow to find $\bar{v}_0, \bar{\sigma}_4$ from all 8 sets of data. To this end one forms 7 combinations $\zeta_D \equiv V_D(r) - d_D V_F(r) = d_D(d_D-1)(\bar{v}_0^{(4)} + \bar{\sigma} r)$. As a typical example one can take fundamental and adjoint potentials,at distances between 0.05fm and 1.1fm from the data [1] the χ^2 fit yields for $\bar{v}_0, \bar{\sigma}_4$

$$\bar{v}_0^{(4)} = (-0.6 \pm 0.67) \cdot 10^{-3} GeV \tag{7}$$

$$\bar{\sigma}^{(4)} = (-1.136 \pm 0.69) \cdot 10^{-3} GeV^2 \tag{8}$$

The quality of the fit is reasonable, $\chi^2/N = 0.45, N = 43$ One obtains similar figures also for D=6,15a,10(while 3 higher representations do not yield additional information)suggesting that $\bar{\sigma}^{(4)}$is negative while $\bar{v}_0^{(4)}$ is compatible

with zero,confirming in this way the parametrization (6). This analysis demonstrates the phenomenon of the Casimir scaling, i.e. proportionality of static potential $V_D(r)$ to the Casimir operator C_D with accuracy better than one percent.

Physical consequences of the Casimir scaling are numerous and important.

First of all, the sign and magnitude of quartic correction (7),(8) can be understood in the FCM. Indeed the quartic term enters the potential V_D with the factor $(-g^4)$, as compared to $+g^2$ for the quadratic (Gaussian) term. Secondly, one can estimate $\langle E_3^2 \rangle$ term from the standard gluonic condensate as follows:

$$g^2 \langle E_3^a E_3^a \rangle \sim \frac{4\pi^2}{12}(0.04 \pm 0.02) GeV^4 \sim (0.10 \pm 0.06) GeV^4 \tag{9}$$

and take into account that the cumulant expansion in (2) is actually in powers of the parameter

$$\xi \equiv g^2 \langle E_3^a E_3^a \rangle T_g^4 \tag{10}$$

Here T_g is the correlation length of the QCD vacuum; for bilocal correlator it was measured on the lattice [6], $T_g^{(2)} \sim 1 GeV^{-1}$. With the use of (10) one could expect that $\bar{\sigma}_4$ would be from 4 to 10% of the standard string tension, $\sigma = 0.2 GeV^2$ provided $T_g = T_g^{(2)}$. The value of $\bar{\sigma}^{(4)}$ calculated in (8) is at least 6 times smaller and suggests that quartic correlation length $T_g^{(4)}$ may be smaller than the Gaussian one, $T_g^{(2)} \sim 0.2 fm$.

This result means that the Gaussian Stochastic Model (GSM), suggested in [3] and successfully used heretofore in many applications [7], can be more accurate than it was even expected, at least in processes where string tension plays the most important role. On the other hand, the smallness of quartic and higher contributions implies a very specific picture of vacuum correlations.

Indeed the smallness of $T_g^{(4)}$ implies that color fields tend to form compact white bilocal combinations $F^a(1)F^a(2)$ which are almost noninteracting between themselves and therefore not contributing to the higher connected correlators. This looks like the picture of small white dipoles made of fields FF (or of vector potentials $A_\mu A_\mu$ connected to FF in the Fock–Schwinger or contour gauge).

One can also understand qualitatively the difference between $T_g^{(2)}$ and $T_g^{(4)}$, since $T_g^{(2)}$ measures correlation length between adjoint fields $E_3^a(x)$ and $E_3^a(y)$ in the Gaussian correlator $\langle E_3^a(x)E_3^a(y)\rangle$, while $T_g^{(4)}$ refers to the correlation of two white complexes, and should be connected to the lowest glueball mass, $M_G \approx 2GeV$; hence $T_g^{(4)} \sim 1/M_G \sim 0.1fm < T_g^{(2)}$.

Finally, the Casimir scaling imposes severe restrictions on existing models of the QCD vacuum. For example, the center–symmetry flux model was tested and ruled out in [1], since in the original formulation it predicts vanishing adjoint string tension, while in the later modification – the fat vertex model [8] it is still far from the accurate data [1].

Next one should mention models of the abelian projected vacuum which fail to provide Casimir scaling [9], at least in the simplest version [10].

Consider now the dilute instanton gas model and the $SU(2)$ group. The instantons may be present in the confining vacuum as an important source of chiral symmetry breaking. Then the Casimir scaling [1] imposes a strict bound on the admixture of instantons in the QCD vacuum. Indeed, insertion in (2) and (4) of the instanton field strength $gF_{34}^a(x,z) = \frac{4\delta_{a3}\rho^2}{[(x-z)^2+\rho^2]^2}$, ($\rho$ – instanton size, z – its position, contribution of parallel transporters is neglected for simplicity since it gives for bilocal correlators a reduction of 20-30%, see [11] for details) yields the following expression for the quartic contribution to the static potential ($r < \rho$) (in $SU(2)$ case)

$$V_D^{(4)}(r \to 0) = -\frac{N}{V}\frac{r^4}{\rho}\,\frac{3C_D^2 - C_D}{15}\frac{\pi^6}{320} \tag{11}$$

where $\frac{N}{V}$ is the density of instanton in the vacuum. Inserting here the values of

$v_0^{(4)}$ above(Eq(7)) one gets the following bounds on the density of instantons

$$\frac{N}{V} \leq 0.17 \; fm^{-4}, \tag{12}$$

which is much smaller than normal instanton density of $1 fm^{-4}$.

With such density the role of instantons in chiral symmetry breaking and other effects would be negligible.

A more stringent bound can be obtained from the quartic string tension generated by instantons . However the nonzero value of $\bar{\sigma}_4$ for instantons does not imply confinement. One should take into account that at large distances the sum of all partial string tensions $\sum_{n=2,4,...} \bar{\sigma}^{(n)}$ for the dilute instanton gas vanishes [12],[13].

In previous discussion we have ignored the fact that at large enough distances the adjoint charges are screened by the vacuum gluons, and the limiting value of the adjoint potential is equal to the doubled gluelump mass, $2M_{gl}$. This leads to an estimate of the screening distance r_0 from the relation; $V_{adj}(r_0) = 2M_{gl}$ where M_{gl} from [14] is around 1.4 GeV and therefore $r_0 \approx 1.4 fm$, which is beyond the distance where Casimir scaling was measured in [1].

Thus the Casimir scaling is a stringent test for all models considered and displays a strong suppression of quartic and higher connected correlators,hence supporting a good accuracy of the GSM. At this point one may wonder how the negligible small higher correlators are compatible with the screening of the adjoint potential at $r \leq r_0$,which is not seen in the cumulant expansion (2).The solution to this problem was suggested in [14], where the screening terms have been identified as an addition to the Wilson loop,with the small coefficient proportional to N_c^{-2}. The corresponding term actually comes from the two- and more Wilson loop averages,and therefore has a perimeter rather than area-law behaviour.Hence sreening cannot be seen in the one-Wilson-loop expansion (2), and the transition due to the properties of the definition (1) of the static potential occurs rather sharply in large T limit.

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